Commutative Property of Addition

Student Probe

Give the student this probe:
What number can you put in this box to make this a true statement?

8 + 4 = ☐ + 3

Typically, a student who does not understand how to read mathematical equations will put 12 or 15. If you further probe and ask, "What does the equal sign mean? Many students will say, "the answer to" or the "total". These students need this lesson.

Lesson Description

Students will learn what the equal sign means by moving through a sequence of true/false statements. They will then solve for the missing variable.

Rationale

The intent is for students to learn to read and interpret mathematical sentences and to begin moving toward relational thinking. This type of "algebraic" thinking along with a fluent ability to interpret mathematical equations will help foster their ability to learn higher level mathematics.

Preparation

Determine the number sentences you want to use with your students and have these available. If giving additional practice, have these sentences prepared to hand students. Initially, use the equations listed in the model lesson. For subsequent lessons some other sets are provided or the teacher can determine other sets of equations to use based on the needs of the students.

At a Glance

What: Learning to comprehend the equal sign in order to read and solve open number sentences in addition and subtraction.
Standard:
AR.Math.Content.1.OA.B.3
Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (*Commutative* property of addition). To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (*Associative* property of addition).
Note: Students need not use formal terms for these properties.
Standards for Mathematical Practices:
SMP2: Reason abstractly and quantitatively.
SMP3: Construct viable arguments and critique the reasoning of others.
SMP4: Model with mathematics.
Who: Students who do not understand the equal sign or the idea of balance on both sides of an equation.
Grade Level: 1
Prerequisite Vocabulary: equal sign, true/false statement
Prerequisite Skills:
One-to-one correspondence, sequencing numbers, numeral recognition, compose and decompose numbers to 20
Delivery Format: This lesson is learned best when there are several students so they discuss together and make meaning through their combined knowledge and processing.
Lesson Length: 15 to 20 minutes at a time
Materials, Resources, Technology: none
Student Worksheets: none

Lesson

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
1. Who knows what this is? (Show a balance scale or a draw a picture of a teeter totter that is balanced or horizontal.) If I put the principal on the right side and a little kindergartener on the left side, what is going to happen?	The side with the principal is going to go down.	Have an actual balance available and put something heavier on one side and lighter on the other to show what happens.
2. Why?	Because he weighs more	What happened to the balance scale? Which side went down? Why?
3. What would happen if I kept putting more first graders on this side until this side weighed the same as the principal?	It would be even.	Have the student add objects until both sides have the same weight and it is balanced.
4. Why?	Because now they are the same weight.	Look at the scale. What do you see about both sides? Why?
5. The equal sign (=) is the same thing as the middle of this balance except when you see the equal sign in a math sentence, it means both sides always have to have the same weight or value.		

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
6. Now, I am going to show you some mathematical statements and I want you to tell me if you think they are true or false. (Uncover the first statement. Keep the others hidden.) 8 + 4 = 12	Yes, because 8 and 4 more makes 12. (If the student doesn't explain, ask, "How do you know?")	Why not? Have the student model with the balance scale and objects.
7. Okay so if that is true, what about this one? 8 + 4 = 12 + 0	Yes, because zero doesn't add anything.	If one student says no, ask the other students what they think. If needed tell the student to model both sides of the number sentence using objects, and ask, "Are they equal?"
8. Okay so if that statement is true, what about this one? 8 + 4 = 0 + 12	Yes, because all you did is change the order of the numbers zero and 12.	I'm not sure.
9. So are you saying that when I am adding 2 numbers together it doesn't matter the order I put them in, the total or sum will always be the same? How do you know? Show me.	Yes, because it is still 12. Student justifies thinking using objects as the model.	
10. Mathematicians call this rule the commutative property. No matter which order you add numbers you will still get the same total.		Have the student model an example with objects. For example, 2+ 3 and 3+2

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
11. Let's try another one 8 + 4 = 8 + 4	Yes, because you have the same numbers on both sides.	Have the student model the equation or have another student convince the other one.
12. So if that is true, what about 8 + 4 = 4 + 8?	Yes, because you just changed the order of the numbers.	Have the student model the equation or have another student convince the other one.
13. So what does the equal sign mean again?	Both sides have to be the same.	
14. Do they have to look the same? Why or why not?	Student gives a previous example when both sides did not look the same.	Have the student look back at previous examples. Were these equal? Do they look the same?
15. Here is another one: 8 + 4 = 9 + 3 Is this a true statement?	Yes, because both sides are 12. OR You just took one from the 4 and added it to the 8 to make 9 + 3.	Have the student model the equation or have another student convince the other one.
16. Here is another statement: 12 = 12	Yes they are the same	
17. What about 12 = 8 + 4	Yes	No you can't write it that way. Ask: what does the equal sign mean? Do both sides have the same value?
18. Now, I am going to give you a statement that has a missing addend, and I want to see if you can complete the sentence: 6 + ☐ = 10	4 How do you know? Because 6 + 4 is 10 and then there is 10 on both sides.	Model the equation.

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
19. Here is another one 6 + 6 = 7 + ☐	5 Because six plus six is twelve and seven and five are twelve. OR Because seven is one more than 6 so since I added one to one 6 I need to take one off the other six which make this number have to be 5. Now they are equal.	Have the student model the equation and determine what is needed and why.
20. Continue in this manner or give the student(s) a worksheet for more practice.		

Teacher Notes

1. It is very easy for students to slip back into thinking the equal sign means "the answer to". They will need lots of practice and discussion to overcome this misconception when reading mathematical sentences.
2. Have the list of number sentences already prepared.
3. Write one sentence at a time as students work through them or have them already prepared and slowly uncover the sentences as you have students work through them.
4. It is important to write the sentences directly under each other so students can make comparisons and build upon what they just discovered.
5. As you use different sets, use different symbols for the variables (?, ☐, ___)
6. For students who have more difficulty, use very small numbers in the beginning (numbers to 5) so they can focus on interpreting the mathematical equation instead of focusing on the computation.
7. Whenever a child uses relational thinking instead of just finding the "answer" to both sides, push this idea by asking "How do you know?" or "Are you saying that I don't have to find the answer, I can just look at the numbers on both sides and see how they are related? Can you model this with objects so we can see how it works?"

Variations

1. Use math squares to find the missing addend or sum. Have students write the equation with the missing variable and then solve.
2. Create sentences with addition on one side and subtraction on the other as an extension.

Formative Assessment

Give the student a series of open number sentences to solve.
What number can you put in this box to make this a true statement?

7 + 6 = ☐ + 5

8 + 4 = 4 + ☐

References

Carpenter, Thomas, P., Franke, Megan Loef, Levi, Linda. *Thinking Mathematically, Integrating Arithmetic and Algebra in Elementary School,* Heinmemann, 2003
Wheatley, Grayson*, Coming to Know Number*, Second Edition, 2010.